



## THE ASSESSMENT OF WATER POLLUTANTS USING GAMMA DISTRIBUTION IN LAGOS STATE, NIGERIA.

**Aaron Akinloye**

Department of Human Kinetics and Health Education  
University of Lagos

E-mail: aakinloye@unilag.edu.ng

**Festus Oladimeji Olafare**

Bilqees Mopelola Oladunni-Mohammad

Department of Technology and Vocational Education  
University of Lagos

E-mail: folafare@unilag.edu.n

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### Abstract

Some stake-holders believe that most health and socio-economic problems in Nigeria are caused by water pollution.

The **main objective** of this study was to examine water pollution in Lagos metropolitan city using a model called gamma distribution. It is of great importance to give attention to water pollution monitoring in the country, considering the rate at which people are being seriously affected by polluted water in the society.

### Methods

Generalized Linear Model (GLM) was used to monitor the concentration of five water pollutants in Lagos State reservoir distribution,

the variables involved are (**turbidity, colour, pH, dissolved oxygen and alkalinity**). W-test was also used to know if the distribution is normal or not

### Results

The average amount of turbidity is 2.6818 mg/l which was higher than the World Health Organisation standard of less than 1mg/l for treated water. The pH of water in the reservoir was 7.49 which conforms with the WHO standard, between 6.5 and 8.5 mg/l. The mean amount of colour in the water was 5.0333 H.U; this is above the WHO standard value of 5.0 HU, while the dissolved oxygen and total alkalinity quantity in the reservoir were 7.5967 mg/l and 38.6833 mg/l respectively, that are in consonance with the WHO standard of portable water quality. **In conclusion**, It was discovered that all the pollutants were significant except the dissolved oxygen. The results presented provide a basis for the use of gamma distribution to model water pollutants. The study therefore recommends that, model used as decision support tools for the management of the Lagos state reservoir, as the attention of government and stake-holders are drawn to keeping water safe.

**Keywords:** Water Pollution, Kolmogorov Test, Gamma Distribution, Estimation

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## **Introduction**

Some stake-holders believe that most health and socio-economic problems in Nigeria are caused by water pollution. It is therefore important to give attention to water pollution monitoring in the country, considering the rate at which people are being seriously affected by polluted water. Water Corporation, one of the major sources of water supply to people of Lagos State, Nigeria is perceived to be undergoing gradual pollution from human related activities at a rate that may constitute health and socio-economic problems to the people. There are many probability distributions that could be successfully utilized to parameterize water pollutants as the critical component of these distributions is their flexibility to represent a variety of events. It should therefore be noted that there is little difference between many of the commonly used distributions when estimating parameters based on limited number of points. One of these available distributions is gamma, making it a good choice for application in this study.

However, a lot of improvement and development has been recorded over decades as probability functions are used in environmental pollution monitoring. Annete (2001) affirmed that, among other probability density functions, those that are extensively used as lognormal, Weibull, beta, gamma, and chi-square distribution. Liu et al. (2003) noted that, the application of water quality model in the white-cart water catchments, Glassglow, UK has received some good results in monitoring the quality of water. Donigian et al. (1981) and Zhang et al. (2004) confirmed the watershed hydraulic processes affecting water quality and offered an overview on the computer models developed and that water quality models with computer. The scale and shape parameters that are common in probability distributions make them to take a variety of shapes and can be used to model both right and left skewed data sets. Weber et al., (1995) carried out a study on statistical methods for assessing ground water compliance and clean-up. They continued by recommending statistical procedures for making regulatory decisions to include hypothesis tests, statistical interval estimates, and trend analyses. Application of these methods within the context of regulatory ground-water monitoring programmes in USA was very much effective. Mu et al, (2004) established a hydrologic statistical model for the river-basin of jialu-River and Juweil



River by applying multiple regression method. In the year 2001, Chen and Yaang, (2001) established an econometric model for urban water pollution loss. The regulations which clearly defined the analysis performance criteria to be met by laboratories carrying out environmental regulatory analysis of pollutants were discussed (Thompson et al., 2006). They focused on the importance of employing model fitting for the purpose of analysis to monitor pollution. Rice et al. (1995), and Wackerly et al. (1996) described in some details, the ideas of estimation and modelling by giving a clear account of basic probability theory, as well as good overview of statistical methods. Chandler et al. (2001) and Gregory et al, (2006) in their study gave an overview of linear model in climate research and the use of the gamma distribution to represent monthly rainfall in Africa for drought monitoring applications demonstrated the feasibility of fitting cell-by-cell probability distributions to grids of monthly interpolated and continent-wide-data. They went further with the discovery that the gamma distribution is suitable for 98% of the locations over all months. The model was used as decision support tools for the management of water, agricultural resources and food reserves. Consequently, an understanding of what statistical method is appropriate, given the type of data that have been collected is clearly important and crucial to realize that different statistical methods have much in common, as one method leads to another. In some cases, it may be necessary to fit different distributions for the data, in a way to see which are best fit by carrying out a goodness of fit test. For instance, the Kolmogorov-Smirnov (Ks) test to unravel the true distribution of the data.

The primary objectives of this paper are to estimate and assess distribution parameters that may be used to describe the amount of pollutants in Water Corporation (Lagos). More specifically, probability distribution parameters are estimated from monthly data on water quality measures collected from the ministry of water resources, Lagos State, and the goodness of fit of the parameters were examined.

## **Materials and Methods**

### **Gamma Distribution**



In probability theory and statistics, the gamma distribution is a two-parameter family of continuous probability distributions. It has a scale and shape parameters, say  $\theta$  and  $K$  respectively. If  $K$  is an integer then the distribution represents the sum of  $K$  independent exponentially distributed random variables, each of which has a mean of  $\theta$  (which is equivalent to a rate parameter of  $\theta^{-1}$  (Wackerly et al., 1996). The gamma distribution function is shown in equation (1), where  $k$  and  $\theta$  represent the shape and scale parameters respectively, as  $x$  represents amount of pollutants.

$$f(x, k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma_k} \text{ for } x > 0 \text{ and } k, \theta > 0 \quad (1)$$

Alternatively, the gamma distribution can be parameterized in terms of a shape parameter  $\alpha = k$  and an inverse scale parameter  $\beta = \frac{1}{\theta}$  called a rate parameter:

$$g(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma \alpha} x^{\alpha-1} e^{-\beta x} \text{ for } x > 0 \quad (2)$$

$$\Gamma \alpha = \int_{-\infty}^{\infty} e^{-x} x^{\alpha-1} dx \quad (3)$$

If the above gamma function is divided by  $\Gamma \alpha$ , it becomes a gamma function distribution.

$$f(x) = \frac{e^{-x} x^{\alpha-1}}{\Gamma \alpha} \alpha > 0; x > 0 \quad (4)$$

This is called a one-parameter case of gamma distribution. However, for a continuous random variable  $x$  such that  $\alpha$  and  $\beta$  are fixed then the two parameter case of gamma distribution is defined in equation (5).

$$f(x) = \frac{e^{-1/\theta} x^{\alpha-1}}{\Gamma \alpha \beta^\alpha} \alpha > 0; \beta > 0; x > 0 \quad (5)$$

The  $r^{\text{th}}$  moment about the origin of equation (5) is obtained as:

$$E(x^r) = \mu_r^1 = \frac{\beta^1 \Gamma(r + \alpha)}{\Gamma \alpha} \quad (6)$$

Setting 'r' equals 1 and 2 in equation (6) gives the mean and variance of the gamma distribution function as obtained in (7) and (8).



$$E(x) = \mu_1^1 = \frac{\beta^1 \Gamma(1 + \alpha)}{\Gamma \alpha}$$

$$E(x) = \frac{\alpha \beta \Gamma \alpha}{\Gamma \alpha}$$

$$E(x) = \alpha \beta \quad (7)$$

Similarly for  $r = 2$

$$E(x^2) = \frac{\beta^2 \Gamma(\alpha + 2)}{\Gamma \alpha}$$

$$E(x^2) = \frac{(\alpha + 1) \Gamma(\alpha + 1)}{\Gamma \alpha} \beta^2$$

$$E(x^2) = \alpha^2 \beta^2 + \alpha \beta^2$$

$$s^2 = E(x^2) - (E(x))^2$$

$$s^2 = \alpha \beta^2 \quad (8)$$

For this study, the gamma distribution parameters are estimated through the maximum likelihood estimation (MLE). Taking the natural log of likelihood function of equation (5), we obtain

$$\begin{aligned} \ln \pi_{i=1}^n f(x) &= \ln e^{\frac{\sum x}{\beta} \pi_x^{\alpha-1}} \frac{1}{\Gamma(\alpha) \beta^{\alpha n}} \\ &= -\frac{\sum x}{\beta} + \ln \pi_{i=1}^n x^{\alpha-1} - \ln \Gamma(\alpha)^n - \ln \beta^{\alpha n} \\ &= \frac{\sum x}{\beta} + (\alpha - 1) \sum \ln x - \ln \Gamma(\alpha)^n - n \alpha \ln \beta \end{aligned} \quad (9)$$

The partial derivative of (6) with respect to  $\beta$  yields its estimate as in (7):

$$\begin{aligned} \frac{\partial \ln \pi^n f(x)}{\partial \beta} &= \frac{\sum x}{\beta^2} - \frac{n \alpha}{\beta} = 0 \\ \Rightarrow \hat{\beta} &= \frac{\bar{x}}{\hat{\alpha}} \quad (10) \end{aligned}$$



Also, differentiating (6) partially with respect to  $\alpha$  giving the estimate in (11):

$$\hat{\alpha} = \frac{1}{4A} \left[ 1 + \sqrt{1 + \frac{4A}{3}} \right] \quad (11)$$

$$\text{where: } A = \ln(\bar{x}) - \frac{\sum_{i=1}^n \ln(x_i)}{n} \quad (12)$$

(Husak et al, 2006).

The calculation of MLEs starts with the derivation of an intermediate value 'A' as shown in equation (12) (Wilks et al., 1995; Ozturk et al., 1981; and Thom et al., 1958), where  $x_i$  equal respective values of pollutants, and the mean ( $\bar{x}$ ) is the arithmetic mean of all pollutant values. Ultimately, 'A' is a measure of the skewness of the distribution, whose value is then used to estimate the shape parameter ' $\alpha$ ' in (11). Interestingly, the product of the shape parameter ( $\alpha$ ) and the square of the scale parameter ( $\beta^2$ ) equals the variance as established in (8).

### Generalized Linear Model (GLM)

The GLM is a flexible generalization of ordinary least square regression. It generalizes linear regression by allowing the linear model to be related to the response variable via a link function, and by allowing the magnitude of the variance of each measurement to be a function of its predicted value (Cullagh et al. 1989).

In this paper, it has been attempted to embed the work and the procedure within the framework of a probability model. In most cases, if there are two  $x$  values that are the same, the associated  $Y$  values will differ. Considering  $Y_i$  as observed value of a random variable  $Y$ , whose distribution depends on  $X$ , an appropriate model for such a situation is:

$$Y_i = \beta_0 + \beta x_i + \epsilon_i$$

Where  $\epsilon_i$  are independent random errors with zero mean and common variance  $\sigma^2$ .

$$\text{Then: } E(Y_i) = \beta_0 + \beta x_i = \mu(x_i), \text{ say}$$



If  $X_i = x_i$ , and we are making extra assumption that the  $\epsilon^1_s$  are normally distributed, then predicting a probability distribution for  $Y$  given  $x$  means that:

$$Y_i \approx (\beta_0 + \beta x_i, \sigma^2) \text{ (Krzonowski et al, 1998 )}$$

The problem considered here is to determine the concentration or amount of pollutants in Lagos reservoir. The quantity of interest is called the dependent variable and other quantities are explanatory variables, predictors, or covariates. The response variable here is Turbidity and the potential predictors are pH, Colour, Dissolved Oxygen and Alkalinity. Using the statistical package 'SPSS' we obtain the GLM for the pollution data, and the results are summarized in table 3.

### Goodness of Fit Test

Having tried to model with gamma distribution, the need to determine the goodness of fits arises. Statistical intervals and test are often on specific distributional assumptions, before computing any analysis on observed data in any area of interest. There is a need to verify if some assumptions are justified, otherwise the correction has to be done on the data after pre-test to make it valid and suitable for intended analysis.

### The W-Test

This test developed by Shapiro and Wilk (1965) is an effective method for testing whether data set have been drawn from an underlying normal distribution. The null hypothesis to be tested is

**H<sub>0</sub>:** The population has a normal distribution.

**H<sub>1</sub>:** The population does not have a normal distribution.

If  $H_0$  is not rejected, then the data set is consistent with the  $H_0$  distribution, although a retest in using additional data could result in rejecting  $H_0$ .

### The Kolmogorov – Smirnov Test



Once the parameters are estimated, their accuracy in approximating the true distribution must be confirmed. To do this, the estimated gamma distribution is compared against the empirical distribution (Ison et al., 1971). This is done with Kolmogorov – Smirnov (KS) goodness of fit test (Crutcher et al., 1975). The Kolmogorov – Smirnov test compares the cumulative distribution functions of the theoretical distribution – the distribution described by the estimated shape and scale parameters – with the observed values and returns the maximum difference between these two cumulative distributions (Wilks et al., 1995). This maximum difference in cumulative distribution functions is frequently referred to as the KS – Statistic.

The acceptable KS value for rejection depends on the number of points in the empirical distribution being used to test the theoretical distribution, and the rejection level chosen for the study (Crutcher et al., 1975). Because the acceptable value of the KS-statistic is a variable, the confidence in accepting or rejecting the theoretical distribution may be measured by the P-value, which incorporates the number of values used in the test into the calculation of its value. Small P-value calls for rejection of the null hypothesis, while a P-value larger than the selected significant level means the null hypothesis cannot be rejected (Wilks et al., 1995).

**Table 1: Estimates of Parameters**

| <b>Probability Distribution</b> | <b>Parameters</b> | <b>Turbidity</b> | <b>Colour</b> | <b>pH</b> | <b>Dissolved Oxygen</b> | <b>Alkalinity</b> |
|---------------------------------|-------------------|------------------|---------------|-----------|-------------------------|-------------------|
| Gamma                           | A                 | 8.6729           | 162.6217      | 199.9002  | 85.6105                 | 19.3603           |
|                                 | B                 | 0.3092           | 0.0309        | 0.0374    | 0.0887                  | 1.9980            |

**Table 2: Summary Table for the Fitted Two-parameter Gamma Distribution**

| <b>Mean and Variance</b> | <b>Turbidity</b> | <b>Colour</b> | <b>Pollutants pH</b> | <b>Dissolved Oxygen</b> | <b>Alkalinity</b> |
|--------------------------|------------------|---------------|----------------------|-------------------------|-------------------|
| $A\beta$                 | 2.6818           | 5.0333        | 7.4900               | 7.5967                  | 38.6833           |
| $\alpha\beta^2$          | 0.8297           | 0.1553        | 0.2796               | 0.6742                  | 77.2941           |



**Table 3: Generalized Linear Model Results**

|     | <b>Coefficients</b> | <b>Standard Error</b> | <b>P-Value</b> | <b>95% Confidence Interval</b> | <b>Remark</b>   |
|-----|---------------------|-----------------------|----------------|--------------------------------|-----------------|
| Col | 0.12952             | 0.24281               | 0.019          | -0.34637, 0.60542              | Significant     |
| pH  | 0.03197             | 0.19134               | 0.026          | -0.34304, 0.40700              | Significant     |
| DO  | 0.08623             | 0.12796               | 0.500          | -0.16457, 0.33704              | Not significant |
| Alk | 0.01187             | 0.01167               | 0.039          | -0.01100, 0.03474              | Significant     |

**Table 4: Shapiro-Wilk Test Result**

| <b>Variable</b>  | <b>Obs</b> | <b>W</b> | <b>V</b> | <b>Z</b> | <b>Prob&gt;z</b> |
|------------------|------------|----------|----------|----------|------------------|
| Turbidity        | 30         | 0.89675  | 5.612    | 3.718    | 0.00010          |
| Colour           | 30         | 0.95278  | 2.567    | 2.032    | 0.02108          |
| pH               | 30         | 0.94523  | 2.977    | 2.352    | 0.00935          |
| Dissolved Oxygen | 30         | 0.85915  | 7.656    | 4.388    | 0.00001          |
| Alkalinity       | 30         | 0.90214  | 5.319    | 3.603    | 0.00016          |

Obs – Observation; W – quantiles of Shapiro-Wilk test for Normality provided in the table ( $W\alpha = 0.967$ ); V – index for departure from normality; Z – test of statistical significance.

**Table 5: Shapiro-Francia Test Result**

| <b>Variable</b>  | <b>Obs</b> | <b>W</b> | <b>V</b> | <b>Z</b> | <b>Prob&gt;z</b> |
|------------------|------------|----------|----------|----------|------------------|
| Turbidity        | 30         | 0.90470  | 5.705    | 3.269    | 0.00054          |
| Colour           | 30         | 0.95893  | 2.458    | 1.729    | 0.04190          |
| pH               | 30         | 0.95199  | 2.874    | 2.020    | 0.02167          |
| Dissolved Oxygen | 30         | 0.85551  | 8.650    | 4.003    | 0.00003          |



|            |    |         |       |       |         |
|------------|----|---------|-------|-------|---------|
| Alkalinity | 30 | 0.89355 | 6.372 | 3.466 | 0.00026 |
|------------|----|---------|-------|-------|---------|

W $\alpha$  – Shapiro-Wilk W test for Normality

**Table 6: Kolmogorov-Smirnov Test Result**

| Parameters | Turbidity | Colour | pH     | Dissolved Oxygen | Alkalinity |
|------------|-----------|--------|--------|------------------|------------|
| D          | 1.0000    | 1.0000 | 1.0000 | 1.0000           | 0.5284     |
| P-value    | 0.100     | 0.070  | 0.100  | 0.080            | 0.074      |

D – the maximum difference between the empirical and theoretical distributions

## Results

### Parameter Estimation

Having obtained the expressions for the shape parameter “ $\alpha$ ” and scale parameter “ $\beta$ ” as in (10) and (11) above using the method of maximum likelihood estimate, the estimates of the two parameters “ $\alpha$ ” and “ $\beta$ ” for each of the water pollutants considered using statistical package “R” are as summarized in Table1.

From table 1 above, we observe that turbidity observation ‘X’ has gamma distribution.

### Fitting Two – Parameter Gamma for Pollutants

With shape parameter  $\alpha = 8.6729$  and scale parameter  $\beta = 0.3092$ , statistically written as: x – gamma ( $\alpha$ ,  $\beta$ )

x – gamma (8.6729, 0.3092)

$$f(x; 8.6729, 0.3092) = \frac{e^{-x} / 0.3092 \times 8.679}{(8.6729) 0.3092^{8.6729}} \quad (13)$$

$$E(x) = \alpha\beta = 2.6818$$

$$v(x) = \alpha\beta^2 = 0.8297$$

Also for colour:  $X \sim \text{gamma}(162.6217, 0.0309)$



$$f(X, 162.6217, 0.0309) = \frac{e^{-x}/0.0309 \times 162.6217}{\Gamma(162.6217)0.0309^{162.6217}} \quad (14)$$

$$E(x) = \alpha\beta = (162.6217)(0.0309)$$

$$E(x) = 5.0333$$

$$V(x) = \alpha\beta^2$$

$$V(x) = (162.6217)0.0309^2 = 0.1553$$

Following the same procedure as above we obtain estimate of means and variances for all other pollutants, and the results are tabulated in table 2.

### Discussion

In table 2, the average amount of turbidity is 2.6818 mg/l which was higher than the World Health Organization (WHO) standard of less than 1mg/l for treated water. Also, the pH of water in the reservoir was 7.49 which conforms with the WHO Standard, between 6.5 and 8.5 mg/l. The mean amount of colour in the water was 5.0333 H.U; this is above the WHO standard value of 5.0 HU, while the Dissolved oxygen and Total Alkalinity quantity in the reservoir were 7.5967 mg/l and 38.6833 mg/l respectively, which are also in consonance with the WHO Standard of portable water quality. However, it is evident here that the reservoir was polluted considering the level of turbidity it contained. It was also observed from the generalized linear model results that the coefficient of all the variables considered are significant except that of Dissolved Oxygen (DO) with P-values greater than 0.05. This means that the dissolved oxygen is the weakest of all the considered pollutants.

More so, fitting the response variable against the independent variables, the model yielded  $R^2$  of 0.4203. This simply means that 42% of the variation in pollution is jointly explained by all the variables (pollutants), an indication of the level of pollution in Water Corporation, informing that the water was fairly polluted. The results obtained from Shapiro-Wilk and Shapiro-Francia are also discussed as shown in tables 4 and 5 above. Since we will reject hypothesis of supporting a gamma distribution at a significant level if W is less than the quantile provided in table (quantiles



of Shapiro-Wilk test for Normality) Values of  $W$ , such that 100P% of the distribution of  $W$  is less than  $W_p$  on table at  $\alpha = 0.05$  i.e.  $W_\alpha = W_{0.05} = 0.967$ .

### Setting

$W_a$  = Shapiro – Wilk statistics for Normality

$W_b$  = Shapiro – Francia statistic for Normality

In the above results, all pollutant values and the response variable have  $W_a$  less than 0.967 (tabulated value). This clearly indicates that the data can arise from another distribution other than normal distribution, and of course buttressing the fact that all the variables are not normally distributed. The general results also follow a strong support for the claim that environmental data are positively skewed, hence the application of the gamma distribution.

Moreover, once the gamma distribution parameters have been estimated, they should be evaluated to understand how accurately they reflect our data, and therefore represent the modeled probability of pollutants in the reservoir. The KS test offers a straightforward method for assessing the relationship between the empirical distribution and the estimated distribution, leading to either acceptance or rejection of the null hypothesis. As used here, the null hypothesis of the KS test is that the pollutants follow gamma distribution with parameters as estimated in equations (10) and (11). The critical region (acceptance or rejection region) is arbitrary, as  $\alpha = 0.05$  was used in this work, meaning that we reject the null hypothesis that the theoretical distribution is performing adequately in modeling the pollutants with p-value less than 0.05. The test produced  $\alpha$  P-value  $> 0.05$  for all the variables; hence, we fail to reject the null hypothesis that the data follow a gamma distribution.

### Conclusion and Recommendations

Summarily, this paper presented estimation of water pollution parameters for the gamma distribution using the maximum likelihood estimates. The ability of the gamma distribution and parameter estimates to adequately fit the empirical distribution of values in the model was tested



using the Kolmogorov Smirnov (KS) test of goodness of fit. The test showed that the gamma distribution and the estimated parameters could not be rejected as a suitable distribution for the water pollution data at  $\alpha$  0.05 level of significance. The hypothesis testing here indicates that the gamma distribution provides a reasonable description and evaluation of pollutants in the reservoir. Through analysis of the distribution parameters, it is possible to examine the amount of pollutant in the Water Corporation and see if it is outside the World Health Organization (WHO 2011) standard for portable water or not. The ultimate application of the information presented in this paper reveals the true value of this research as we conclude that the Water Corporation is polluted and we recommend that:

- Government should set up a committee, which will ensure that statistically verifiable ideal standards are maintained before the water is treated and passed out to the people for drinking.
- Waste organic materials generated from the plant (company) which are washed into the drain should be reduced so that the Dissolve Oxygen will rise and survival of aquatic life will be better, and
- Regular environmental audit on Water Corporation is done, so that government will ascertain the level of pollutants in the effluents as compared with the existing standards.



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