



## On the Metric-Location-Domination Number of Some Exponential Graphs

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**Abstract**—If  $u$  and  $v$  are vertices in a simple and undirected graph. A distance  $d(u, v)$  is the shortest path between  $u$  and  $v$  in  $G$ . For an ordered set  $W = \{w_1, w_2, \dots, w_k\}$  of vertices and a vertex  $v \in G$ , the representation of  $v$  with respect to  $W$  is the ordered  $k$ -tuple  $r(v|W) = \{d(v, w_1), d(v, w_2), \dots, d(v, w_k)\}$ . A dominating set  $W$  in a connected graph  $G$  is a metric-locating-dominating set (MLD), if  $r(v|W)$  for  $v \in V(G)$  are distinct. The metric-location-domination number of  $G$  denotes  $\gamma_M(G)$  is the minimum cardinality of an MLD-set in  $G$ . The purpose of this paper is to find the metric-location-domination number of some exponential graphs. By an exponential graph, we mean a graph formed by combining two graphs  $G$  and  $H$ , where each edge of graph  $G$  is replaced by the graph  $H$ , denote by  $G^H$ . We have determined the metric-location-domination number of some exponential graph, namely  $P_n^{Btm}$  and  $P_n^{Wm}$ .

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**Index Terms:** metric-locating-dominating, metric-location-domination number, exponential graph.

### I. INTRODUCTION

Graph theory is one part of mathematics were very useful to help solve a problem in life. If a problem in life represented in graph it will be easier understandable and simpler, so it is easier to solve. For example, shortest path in transport problems that can represented in the form of graphs (Chartrand dan Lesniak [3]).

A graph  $G$  is a finite nonempty set of  $V(G) = \{v_1, v_2, \dots, v_n\}$  called vertex and  $E(G) = \{e_1, e_2, \dots, e_n\}$  is every pair sets of  $V(G)$  called edge. Member of the vertices  $V(G)$  (cardinality), denoted by  $|V(G)|$ , called the order of the graph  $G$ . A graph  $G$  called a simple graph if each side of graph  $G$  connecting two different vertices and each of the two

vertices different in the graph  $G$  is only connected by one edge. A Graph  $G$  connected if any two vertices  $u$  and  $v$  in the graph  $G$  there is always a path that sequence of vertex and edge which connects  $u$  to  $v$ .

The distance between two vertices  $u$  and  $v$   $d_G(u, v)$  in a connected graph  $G$  is the length of the shortest path from  $u$  to  $v$  in  $G$ . Representations of  $v$  to  $W$  in  $G$  is a vector with  $k$  tuple  $r(v|W) = (d(v, w_1), d(v, w_2), \dots, d(v, w_k))$  to its components is the distance of  $v$  to all vertices in  $W$ . The set  $W$  is called a locating set if  $r(u|W) = r(v|W)$  then  $u = v$  for  $u, v \in G$ . Furthermore, a set  $W \subseteq V(G)$  is a dominating set of  $G$  if every vertex in  $V(G) - W$  is adjacent to a vertex of  $W$ . The domination number  $\gamma(G)$  is the minimum cardinality of a dominating set of  $G$ . A dominating set of cardinality  $\gamma(G)$  is called a  $\gamma(G)$ -set. The literature on this subject has been surveyed and detailed in the two books by Haynes et.al [8] and [9].

The concepts of a locating set and a dominating set merged by defining the metric-locating-dominating set, denoted by an MLD- set, in a connected graph  $G$  to be a set of vertices of  $G$  that is both a dominating set and a locating set in  $G$  (Henning and Oellermann [7]). Henning and Oellermann [7] define the metric-location-domination number  $\gamma_M(G)$  of  $G$  to be the minimum cardinality of an MLD-set in  $G$ . An MLD-set in  $G$  of cardinality  $\gamma_M(G)$  is called a  $\gamma_M(G)$ -set.

The study and development of the concept MLD-set continued among others by Iswadi cite Isw11a, cite Isw11b, C 'a ceres et al cite Cac13, and Palupi cite Pal13 concept MLD-set still studied and developed. Such research become a reference to search for



metric location domination number of other graph. By an exponential graph, we mean a graph formed by combining two graphs  $G$  and  $H$ , where each edge of graph  $G$  is replaced by the graph  $H$ , denote by  $G^H$ . a  $P_n^{Btm}$  graph and  $P_n^{Wm}$  graph is the one that has not been studied its metric location domination number.

## II. THE RESULT

Suppose  $G$  is a graph with vertices  $v$  and edge  $e$ , where each edge  $e$  of a graph  $G$  is an exponential graph of another graph, for example a graph  $H$ . By an exponential graph, we mean a graph formed by combining two graphs  $G$  and  $H$ , where each edge of graph  $G$  is replaced by the graph  $H$ , denote by  $G^H$ .

A  $P_n^{Btm}$  graph where  $n \geq 2, m \geq 2$  is an exponential graph with the vertices and the edge respectively,

$$\begin{aligned} V &= \{x_i | 1 \leq i \leq n\} \cup \{x_{i,j} | 1 \leq i \leq n-1, \\ &\quad 1 \leq j \leq m\} \\ E &= \{x_i x_{i+1} | 1 \leq i \leq n-1\} \cup \{x_i x_{i,j} | \\ &\quad 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \\ &\quad \{x_{i+1} x_{i,j} | 1 \leq i \leq n-1, 1 \leq j \leq m\}, \end{aligned}$$

and have  $|V| = mn - m + n$  and  $|E| = 2mn - 2m - 1$ .

**Teorema 1.** If  $P_n^{Btm}$  is the exponential graph where  $n \geq 2, m \geq 2$  then,

- 1)  $\gamma_M(P_n^{Btm}) = mn - m, n = 2, 3$  and  $m \geq 2$
- 2)  $\gamma_M(P_n^{Btm}) = mn - m - n + 1 + \lfloor \frac{n}{3} \rfloor, n \geq 4$  and  $m \geq 2$ .

**Proof.** Metric location domination number of  $P_n^{Btm}$  graph consists of two cases, that is  $n = 2, 3$  and another  $n$  ( $n \geq 4$ ).

**Case 1.** Let  $n = 2, 3$  dan  $m \geq 2$ . If we choose  $W = \{x_{i+1} | 1 \leq i \leq n-1\} \cup \{x_{i,j} | 1 \leq i \leq n-1, 1 \leq j \leq m-1\}$ . Then  $W$  is dominator because of every vertex  $V(P_n^{Btm}) - W$  adjacent with a vertex from  $W$ . Obtained representation of any vertices of the  $P_n^{Btm}$  graph with respect to  $W$  are distinct, namely,

$$\begin{aligned} r(x_i | W) &= \{(a_{i,k}), 1 \leq i \leq n, 1 \leq k \leq \\ &\quad mn - m\} \\ r(x_{i,j} | W) &= \{(a_{i,j,k}), 1 \leq i \leq n, 1 \leq j \leq m, \\ &\quad 1 \leq k \leq mn - m\} \end{aligned}$$

where (i)  $(a_{i,k}) = |k + 1 - i|$  for  $1 \leq i \leq n, 1 \leq k \leq n-1$ ; (ii)  $(a_{i,k}) = \left\lceil \frac{k+1-n}{m-1} \right\rceil - i + 1$  for  $1 \leq i \leq \left\lceil \frac{k+1-n}{m-1} \right\rceil, n-1 < k \leq mn-m$ ; (iii)  $(a_{i,k}) = i - \left\lceil \frac{k+1-n}{m-1} \right\rceil$  for  $\left\lceil \frac{k+1-n}{m-1} \right\rceil > i \geq n, n-1 < k \leq mn-m$ . Furthermore, (i)  $(a_{i,j,k}) = k + 1 - i$  for  $1 \leq i < k+1, 1 \leq j \leq m, 1 \leq k \leq (n-1)$ ; (ii)  $(a_{i,j,k}) = i - k$  for  $k+1 \leq i \leq n, 1 \leq j \leq m, 1 \leq k \leq (n-1)$ ; (iii)  $(a_{i,j,k}) = 0$  for  $i = \left\lceil \frac{k+1-n}{m-1} \right\rceil, j \cong (k+1-n) \pmod{m-1}, n-1 < k \leq mn-m$ ; (iv)  $(a_{i,j,k}) = 2$  for  $i, j$ , and  $k$  otherwise

Consequently  $W$  is MLD-set with  $nm - m$  elements. In the case  $n = 2, 3$  not found a MLD-set with elements less than  $mn - m$ . Thus  $\gamma_M(P_n^{Btm}) = mn - m$ , for  $n = 2, 3$ , and  $m \geq 2$ .

**Case 2.** Suppose  $n$  otherwise ( $n \geq 4$ ) and  $m \geq 2$ . For  $n \cong 5 \pmod{3}, n \cong 6 \pmod{3}$ , if we choose  $W = \{x_{3i-1} | 1 \leq i \leq \lfloor \frac{n}{3} \rfloor\} \cup \{x_{i,j} | 1 \leq i \leq n-1, 1 \leq j \leq m-1\}$ . Then  $W$  is dominator because of every vertex  $V(P_n^{Btm}) - W$  adjacent with a vertex from  $W$ . Obtained representation of any vertices of the  $P_n^{Btm}$  graph with respect to  $W$  are distinct, namely

$$\begin{aligned} r(x_i | W) &= \{(a_{i,k}), 1 \leq i \leq n, 1 \leq k \leq \\ &\quad mn - m - n + 1 + \lfloor \frac{n}{2} \rfloor\} \\ r(x_{i,j} | W) &= \{(a_{i,j,k}), 1 \leq i \leq n, 1 \leq j \leq m, \\ &\quad 1 \leq k \leq mn - m - n + 1 + \lfloor \frac{n}{2} \rfloor\} \end{aligned}$$

Where (i)  $(a_{i,k}) = |3k - 1 - i|$  for  $1 \leq i \leq n, 1 \leq k \leq \lfloor \frac{n}{3} \rfloor$ ; (ii)  $(a_{i,k}) = \left\lceil \frac{k - \lfloor \frac{n}{3} \rfloor}{m-1} \right\rceil - i + 1$  for  $1 \leq i \leq \left\lceil \frac{k - \lfloor \frac{n}{3} \rfloor}{m-1} \right\rceil, k > \lfloor \frac{n}{3} \rfloor$ ; (iii)  $(a_{i,k}) = i - \left\lceil \frac{k - \lfloor \frac{n}{3} \rfloor}{m-1} \right\rceil$ , for  $\left\lceil \frac{k - \lfloor \frac{n}{3} \rfloor}{m-1} \right\rceil > i \geq n, k > \lfloor \frac{n}{3} \rfloor$ . Furthermore (i)  $(a_{i,j,k}) = 3k - 1 - i$  for  $1 \leq i < 3k, 1 \leq j \leq m, 1 \leq k \leq \lfloor \frac{n}{3} \rfloor$ ; (ii)  $(a_{i,j,k}) = i - 3k + 2$  for  $3k \leq i \leq n, 1 \leq j \leq m, 1 \leq k \leq \lfloor \frac{n}{3} \rfloor$ ; (iii)  $(a_{i,j,k}) = i = \left\lceil \frac{k - \lfloor \frac{n}{3} \rfloor}{m-1} \right\rceil, j \cong (k - \lfloor \frac{n}{3} \rfloor) \pmod{m-1}, k > \lfloor \frac{n}{3} \rfloor$ ; (iv)  $(a_{i,j,k}) = 2$  for  $i = \left\lceil \frac{k - \lfloor \frac{n}{3} \rfloor}{m-1} \right\rceil, j \not\cong (k - \lfloor \frac{n}{3} \rfloor) \pmod{m-1}, k > \lfloor \frac{n}{3} \rfloor$ ; (v)  $(a_{i,j,k}) = \left| i - \left\lceil \frac{k - \lfloor \frac{n}{3} \rfloor}{m-1} \right\rceil \right|$  for  $\left| i - \left\lceil \frac{k - \lfloor \frac{n}{3} \rfloor}{m-1} \right\rceil \right|, i \neq \left\lceil \frac{k - \lfloor \frac{n}{3} \rfloor}{m-1} \right\rceil, 1 \leq j \leq m, k > \lfloor \frac{n}{3} \rfloor$ .

Representation of  $V(P_n^{Btm})$  with respect to  $W$  are distinct. So we have  $W$  is MLD-set with  $mn - n - m + 1 + \lfloor \frac{n}{2} \rfloor$  elements. in the case  $n \geq$  not found a MLD-set with less elements. Thus  $\gamma_M(P_n^{Btm}) = mn - n - m + 1 + \lfloor \frac{n}{2} \rfloor$ , for  $n \geq 4$ , and  $m \geq 2$



From **Case 1.** and **Case 2.** proved that  $\gamma_M(P_n^{Btm}) = mn - m$ , for  $n = 2, 3$ , and  $m \geq 2$ , otherwise for  $n \geq 4, m \geq 2$  we have  $\gamma_M(P_n^{Btm}) = mn - m - n + 1 + \lfloor \frac{n}{2} \rfloor$ .  $\square$

Furthermore, a  $P_n^{Wm}$  graph where  $n \geq 2, m \geq 3$  is an exponential graph with the vertices and the edge respectively,

$$\begin{aligned} V &= \{x_{i,j} | 1 \leq i \leq n-1, 1 \leq j \leq m-1\} \cup \{x_{n,m}\} \cup \{y_i | 1 \leq i \leq n-1\} \\ E &= \{x_{i,1}x_{i+1,1} | 1 \leq i \leq n-2\} \cup \{x_{n-1,1}x_{n,m}\} \cup \{x_{i,j}x_{i,j+1} | 1 \leq i \leq n-1, 1 \leq j \leq m-2\} \cup \{x_{i+1,1}x_{i,m-1} | 1 \leq i \leq n-2\} \cup \{x_{n,m}x_{n-1,m-1}\} \cup \{y_i x_{i,j} | 1 \leq i \leq n-1, 1 \leq j \leq m-1\} \cup \{y_i x_{i+1,1} | 1 \leq i \leq n-2\} \end{aligned}$$

and have  $|V| = mn - m + 1$  and  $|E| = 2mn - 2m$ .

**Teorema 2.** If  $P_n^{Wm}$  is the exponential graph where  $n \geq 2, m \geq 3$  then,

- 1)  $\gamma_M(P_n^{Wm}) = n - 1$  for  $n \geq 2$  and  $m = 3, 4$ ;
- 2)  $\gamma_M(P_n^{Wm}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor$  for  $n = 2$  and  $m \geq 5$ ;
- 3)  $\gamma_M(P_n^{Wm}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor + (n-3)\lfloor \frac{m-4}{2} \rfloor$  for  $n \geq 3$  and  $m \geq 5$ .

**Proof.** Metric location domination number of  $P_n^{Wm}$  graph consists of three cases,

**Case 1.** For  $n \geq 2$  and  $m = 3, 4$ . If we choose  $W = \{y_i | 1 \leq i \leq n-1\}$ , then  $W$  is dominator because of every vertex  $V(P_n^{Wm}) - W$  adjacent with a vertex from  $W$ . Obtained representation of any vertices of the  $P_n^{Wm}$  graph with respect to  $W$  are distinct. Consequently  $W$  is MLD-set with  $n-1$  elements. In the case  $n \geq 2$  and  $m = 3, 4$  not found a MLD-set with less elements. Thus  $\gamma_M(P_n^{Wm}) = n - 1$ .

**Case 2.** For  $n = 2$  and  $m \geq 5$ . If we choose  $W = \{y_i | 1 \leq i \leq n-1\} \cup \{x_{1,2j} | 1 \leq j \leq \lfloor \frac{m-3}{2} \rfloor\} \cup \{x_{n-1,2j+1} | 1 \leq j \leq \lfloor \frac{m-3}{2} \rfloor\}$ , then  $W$  is dominator because of every vertex  $V(P_n^{Wm}) - W$  adjacent with a vertex from  $W$ . Obtained representation of any vertices of the  $P_n^{Wm}$  graph with respect to  $W$  are distinct. Consequently  $W$  is MLD-set with  $n - 1 + 2\lfloor \frac{m-3}{2} \rfloor$  elements. In the case  $n = 2$  and

$m \geq 5$  not found a MLD-set with less elements. Thus  $\gamma_M(P_n^{Wm}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor$ .

**Case 3.** For  $n \geq 3$  and  $m \geq 5$ . If we choose  $W = \{y_i | 1 \leq i \leq n-1\} \cup \{x_{1,2j} | 1 \leq j \leq \lfloor \frac{m-3}{2} \rfloor\} \cup \{x_{i,2j+1} | 2 \leq i \leq n-2, 1 \leq j \leq \lfloor \frac{m-4}{2} \rfloor\} \cup \{x_{n-1,2j+1} | 1 \leq j \leq \lfloor \frac{m-3}{2} \rfloor\}$ , then  $W$  is dominator because of every vertex  $V(P_n^{Wm}) - W$  adjacent with a vertex from  $W$ . Obtained representation of any vertices of the  $P_n^{Wm}$  graph with respect to  $W$  are distinct. Consequently  $W$  is MLD-set with  $n - 1 + 2\lfloor \frac{m-3}{2} \rfloor + (n-3)\lfloor \frac{m-4}{2} \rfloor$  elements. In the case  $n \geq 3$  dan  $m \geq 5$  not found a MLD-set with less elements. Thus  $\gamma_M(P_n^{Wm}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor + (n-3)\lfloor \frac{m-4}{2} \rfloor$ . From **Case 1.**, **Case 2.**, and **Case 3.** proved that for  $n \geq 2, m = 3, 4$  then  $\gamma_M(P_n^{Wm}) = n - 1$ . For  $n = 2, m \geq 5$  then  $\gamma_M(P_n^{Wm}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor$ . For  $n \geq 3, m \geq 5$  then  $\gamma_M(P_n^{Wm}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor + (n-3)\lfloor \frac{m-4}{2} \rfloor$ .  $\square$

### III. CONCLUSION

We have shown the existence of Metric-Location-Domination Number of some exponential graph, namely  $P_n^{Btm}$ ,  $P_n^{Wm}$  and  $C_n^{Btm}$  graphs. We can prove that (i)  $\gamma_M(P_n^{Btm}) = mn - m$ , for  $n = 2, 3; m \geq 2$  and  $\gamma_M(P_n^{Btm}) = mn - m - n + 1 + \lfloor \frac{n}{2} \rfloor$ , for  $n \geq 4, m \geq 2$ . Thus, we propose the following open problems. (ii)  $\gamma_M(P_n^{Wm}) = n - 1$  for  $n \geq 2$  and  $m = 3, 4$ ;  $\gamma_M(P_n^{Wm}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor$  for  $n = 2$  and  $m \geq 5$ ; and  $\gamma_M(P_n^{Wm}) = n - 1 + 2\lfloor \frac{m-3}{2} \rfloor + (n-3)\lfloor \frac{m-4}{2} \rfloor$  for  $n \geq 3$  and  $m \geq 5$ .

**Open Problem 1.** Find the metric-location-domination number in the other graph.

**Open Problem 2.** Let  $P_n$  be a line graph and  $H$  be any graph. How many  $\gamma_M(P_n^H)$ .

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